4.2 - Translations of Sine and Cosine

Warmup

Find the exact value.

1) $\dfrac{\sin 30^\circ + \cos 60^\circ}{2}$

$\dfrac{1}{2}$

2) $\dfrac{4\sin 300^\circ + 2\cos 30^\circ}{3}$

$-\dfrac{\sqrt{3}}{3}$

3) $4(\sin 30^\circ)(\cos 60^\circ)$

1

4) $\sin 30^\circ + \sin 60^\circ$

$\dfrac{1+\sqrt{3}}{2}$

5) $(\sin 60^\circ)^2 + (\cos 60^\circ)^2$

1

6) $8(\sin 120^\circ)(\cos 120^\circ)$

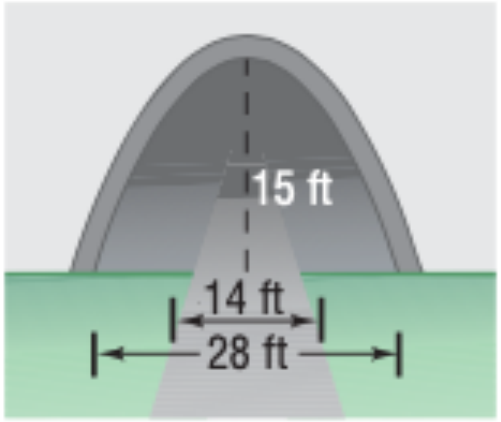
$-2\sqrt{3}$

Ch 4.1 Word Problems

4.1 - Graphing Sine and Cosine

Sample Problem

92. Bridge Clearance A one-lane highway runs through a tunnel in the shape of one-half a sine curve cycle. The opening is 28 feet wide at road level and is 15 feet tall at its highest point.



$$h = ?\sin(?x) + ?$$

$$h = 15\sin(\frac{\pi}{28}x) + 0$$

(a) Find an equation for the sine curve that fits the opening. Place the origin at the left end of the sine curve.

(b) If the road is 14 feet wide with 7-foot shoulders on each side, what is the height of the tunnel at the edge of the road?

$$h = 15\sin(\frac{2\pi}{56} \cdot 7) + 0$$

$$h = 10.6\ ft$$

4.1 - Graphing Sine and Cosine

Practice

Motion of a Buoy A signal buoy in the Chesapeake Bay bobs up and down with the height h of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants a and b?

$$h = ?\sin(?x) + 5$$

$$h = 4\sin(\frac{4\pi}{7}t) + 5$$

4.1 - Graphing Sine and Cosine

Practice

Motion of a Buoy A signal buoy in the Chesapeake Bay bobs up and down with the height h of its transmitter (in feet) above sea level modeled by $h = a \sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft height to the next. What are the values of the constants a and b?

a) What is the height after 2.5 seconds?

$h = 1.1\ ft$

b) How long before the height is 8 feet the first two times?

$t = 0.47\ secs,\ 1.28\ secs$

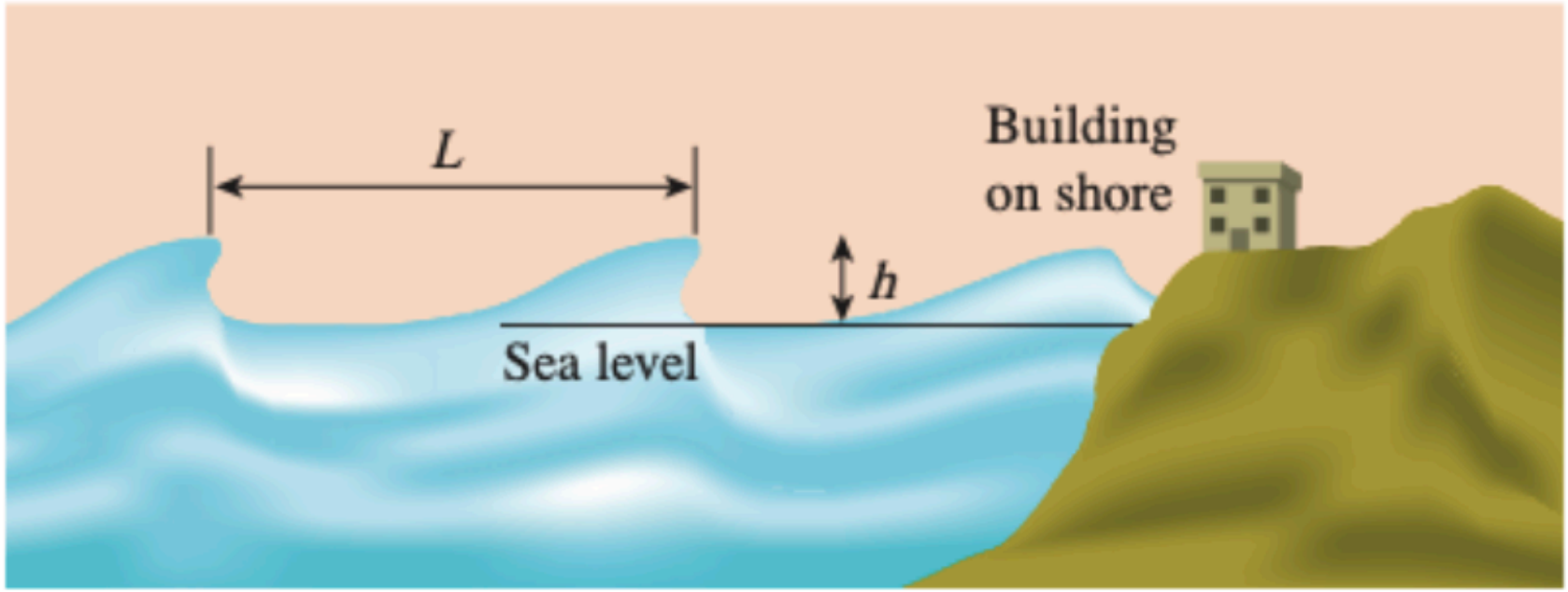
4.1 - Graphing Sine and Cosine

Practice

5/11

Tsunami Wave An earthquake occurred at 9:40 AM on Nov. 1, 1755, at Lisbon, Portugal, and started a tsunami (often called a tidal wave) in the ocean. I produced sinusoidal waves that reached a height of 60 ft.

a) If the period of the waves was 1800 sec (30 min), find the equation of the height relative to the time.



$$h = 30 \sin\left(\frac{2\pi}{1800}t\right) + 30$$

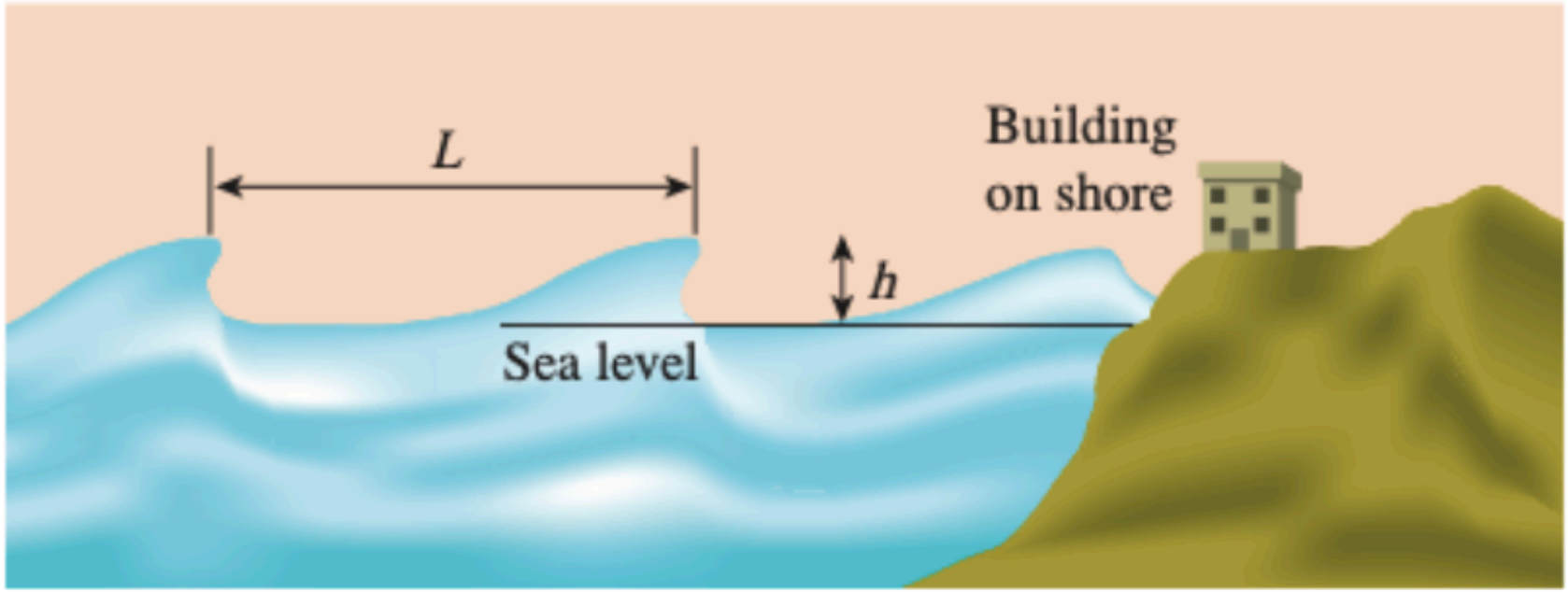
4.1 - Graphing Sine and Cosine

Practice

9/11

Tsunami Wave An earthquake occurred at 9:40 AM on Nov. 1, 1755, at Lisbon, Portugal, and started a tsunami (often called a tidal wave) in the ocean. I produced sinusoidal waves that reached a height of 60 ft.

b) The wave traveled a distance of 1000 ft. onshore each period. How long (hrs) did it take to travel 5280 ft. (1 mile)?



$$h = 30\sin\left(\frac{2\pi}{1800}t\right) + 30$$

$$5280\ ft \cdot \frac{0.5 hours}{1000\ ft} = 2.64 hrs$$

4.1 - Graphing Sine and Cosine

Practice

Ferris Wheel A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground find the equation of the height relative to the time.

$$h = -25\cos\left(\frac{2\pi}{40}t\right) + 30$$

$$h = 25\sin\left(\frac{2\pi}{40}(t-10)\right) + 30$$

4.1 - Graphing Sine and Cosine

Practice

Ferris Wheel A Ferris wheel 50 ft in diameter makes one revolution every 40 sec. If the center of the wheel is 30 ft above the ground, how long after reaching the low point is a rider 50 ft above the ground?

$$h = -25\cos\left(\frac{2\pi}{40}t\right) + 30$$

$$h = 25\sin\left(\frac{2\pi}{40}(t-10)\right) + 30$$

$t = 15.9\ sec,\ 24.1\ sec$

4.2 - Translations of Sine and Cosine

Warmup

Find the exact value.

1) $\dfrac{\sin 1830^\circ + \cos(-1020^\circ)}{2}$

$\dfrac{1}{2}$

2) $\dfrac{4\sin(\frac{5\pi}{3}) + 2\cos 5070^\circ}{3}$

$-\dfrac{\sqrt{3}}{3}$

3) $\sin(-\dfrac{11\pi}{6}) + \sin(-300^\circ)$

$\dfrac{1+\sqrt{3}}{2}$

4) $8(\sin\dfrac{8\pi}{3})(\cos(-240^\circ))$

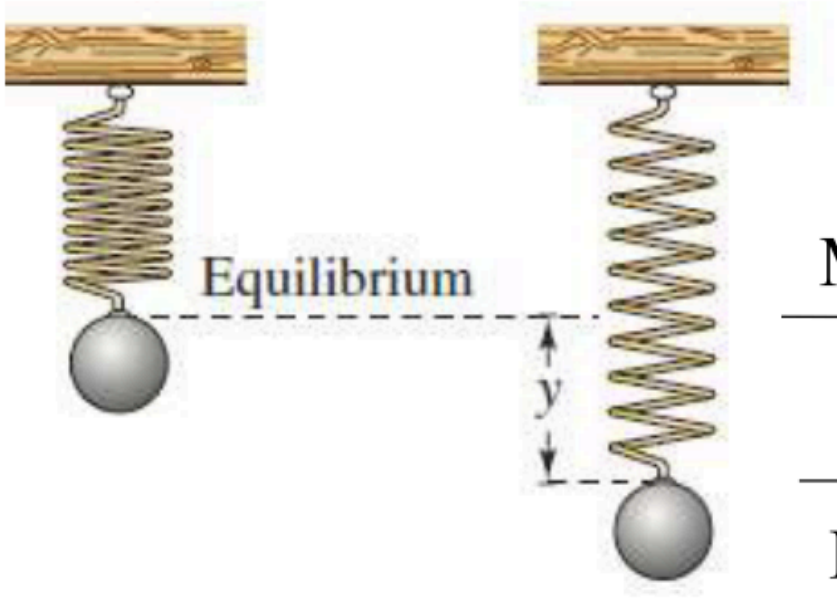
$-2\sqrt{3}$

4.1 - Graphing Sine and Cosine

8/11

Practice

A mass on a spring is pulled down and then released. What is the equation of the height? One cycle is finished in 1.2 seconds.



Max = 10 in.

Min = 0 in.

$h = ?\cos(?x) + ?$

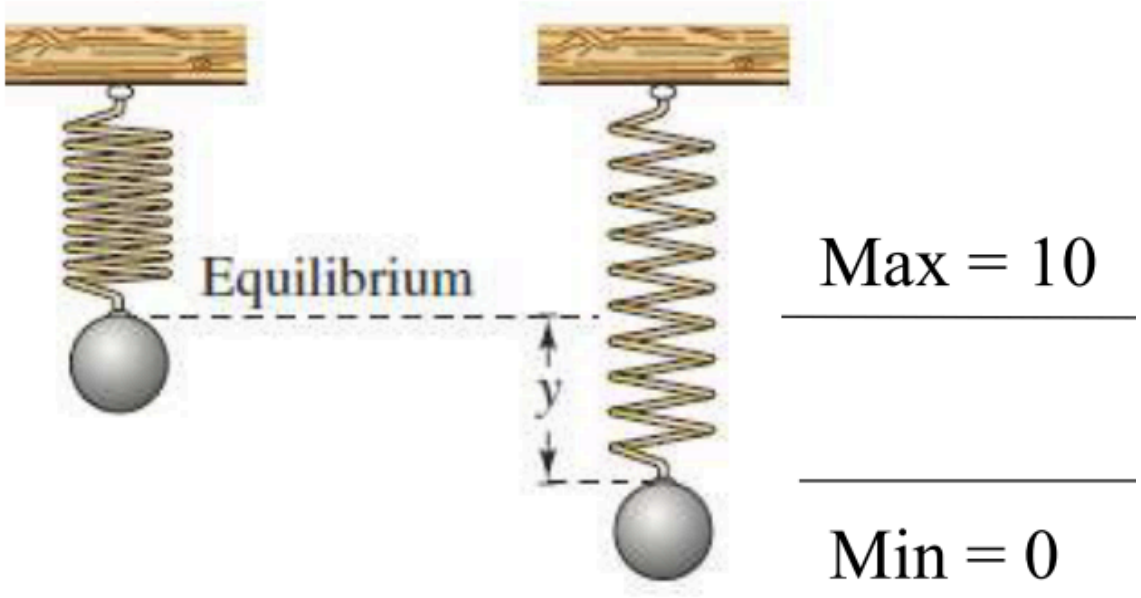
$h = -5\cos(\frac{5\pi}{3}t) + 5$

4.1 - Graphing Sine and Cosine

9/11

Practice

A mass on a spring is pulled down and then released. Within the first 5 seconds, how many times has it reached a height of 1.0 inches?



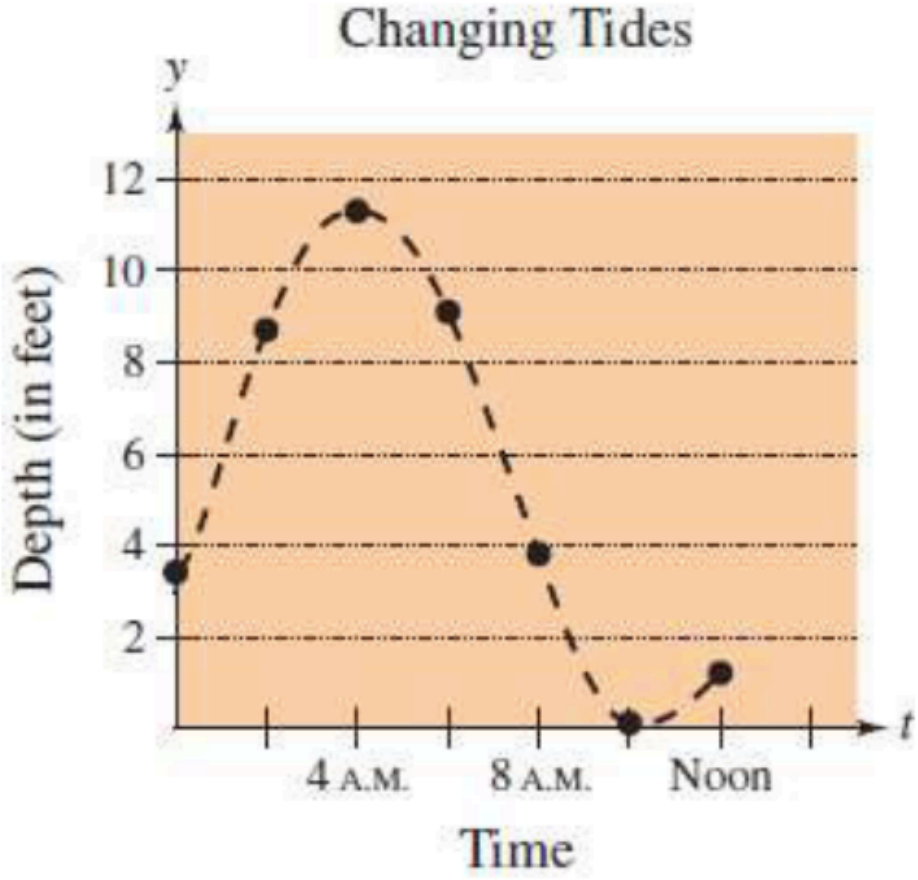
$$h = -5\cos\left(\frac{5\pi}{3}t\right) + 5$$

$4 * 2 + 1$ *or* 9 *times*

4.1 - Graphing Sine and Cosine

Practice

Find the equation of the tide height given the graph. Use a sine function to model the height. (Max = 11 ft)



$$h = 5.5 \sin\left(\frac{2\pi}{14}(t-1)\right) + 5.5$$

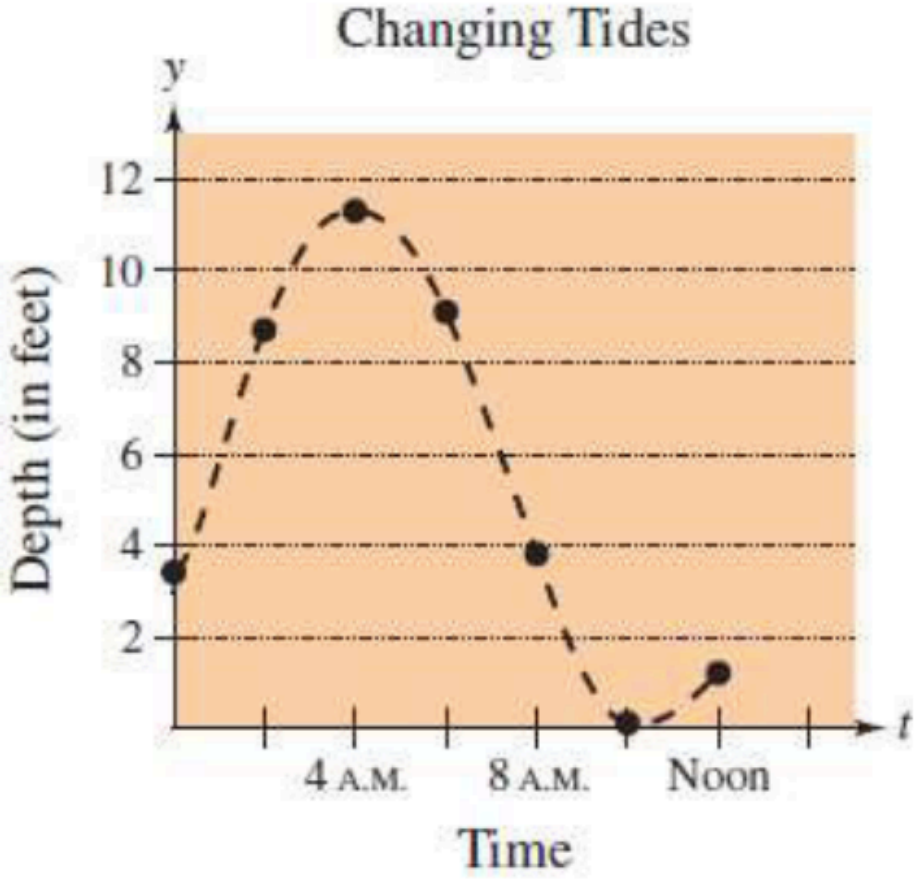
4.1 - Graphing Sine and Cosine

Practice

What is the tide height at 2 PM?

$$h = 5.5 \sin\left(\frac{2\pi}{14}(t-1)\right) + 5.5$$

$h \approx 6.1\ ft$

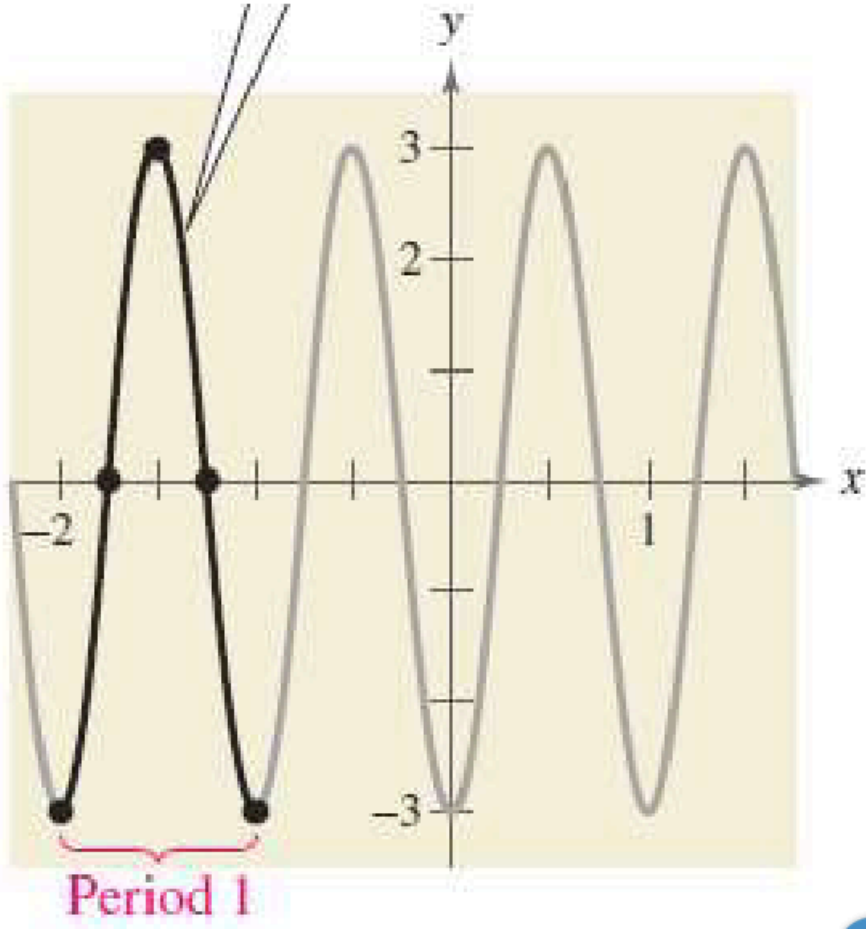


4.1 - Graphing Sine and Cosine

Practice

Given the graph on the right, find the cosine function. The dark portion was originally $[0, 2\pi)$. Only reflection on X and any translation is allowed.

$$y = -3\cos\left(2\pi(x+2)\right)$$



4.1 - Graphing Sine and Cosine

11/11

Practice

One particular July 4th in Galveston, TX, high tide occurred at 9:36 A.M. At that time the water at the end of the 61st Street Pier was 2.7 meters deep. Low tide occurred at 3:48 P.M., at which time the water was only 2.1 meters deep. Assume that the depth of the water is a sinusoidal function of time with a period of half a lunar day (about 12 hours 24 minutes).

Find the cosine equation to represent this function of depth d(t) (in meters) versus time (in hours).

$$d(t) = 0.3\cos(\frac{\pi}{6.2}(t - 9.6)) + 2.4$$